A New Technique for the Measurement of Spinwave Linewidths from the Saturation of the Ferrimagnetic Resonance Line*

P. E. SEIDEN†, MEMBER, IRE

Summary—A new measurement technique is described that records the ferrimagnetic saturation curve in a way that permits the Suhl critical field to be evaluated directly from the recording. This technique eliminates the tedious task of constructing point by point saturation curves for evaluating the critical field and shortens immensely the time required to determine spinwave linewidths from the saturation of the ferrimagnetic resonance line.

I. Introduction

Since 1956, when Suhl[1] explained the premature ferrimagnetic resonance saturation effects found by Damon[2] and Bloembergen and Wang,[3] it has been obvious that experiments on these effects could yield information on the loss parameters of the spinwaves excited, thus shedding some light on the relaxation processes occurring in ferrimagnets. The saturation of the ferrimagnetic resonance line is caused by the fact that at large enough precession angles the uniform precession transfers energy to a degenerate spinwave k ($\omega_0=\omega_k$, $k\neq 0$) that propagates along the direction of the dc magnetic field. The linewidth of this spinwave (ΔH_k) may be obtained from a measurement of the threshold RF magnetic field (h_c) at which the susceptibility begins to decline due to this effect. This field is related to ΔH_k by

$$h_c = \Delta H \sqrt{\frac{\Delta H_k}{4\pi M_s}} \tag{1}$$

where ΔH is the small signal uniform precession linewidth and $4\pi M_s$ is the saturation magnetization. According to the theory of this effect, the threshold for saturation should be sharply defined as shown in Fig. 1; however, it is experimentally observed (Fig. 2) that this is not the case. Schlömann[4] has explained the gradual onset of saturation in terms of inhomogeneity scattering in the material which linearly excites degenerate spinwaves. Therefore, the number of spinwaves k at threshold will be considerably above the thermal level.

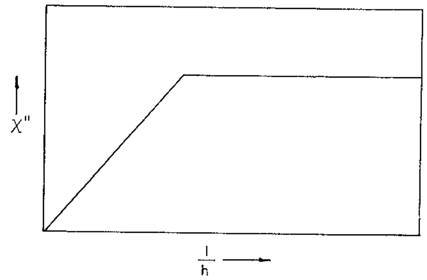

Fig. 1—Theoretical saturation curve for the ferrimagnetic resonance line.

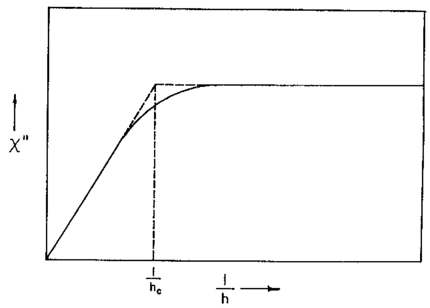

Fig. 2—Experimentally observed saturation behavior (schematic).

This lack of sharpness of the threshold has necessitated the development of an alternative method of determining h_c.[4-6] The nonlinear theory of saturation predicts a susceptibility that varies as $1/h$ above threshold. Since reasonably above threshold we have a spinwave population large compared to that due to the inhomogeneity scattering effect, we may assume that the observed $1/h$ behavior in this region is due to the nonlinear effect only and extrapolate this behavior back to $\chi''/\chi_0''=1$ to get the threshold field[5] (Fig. 2). The difficulty with this technique is that it is rather time-consuming, since one must make a point by point measurement of the saturation curve, calculate χ''/χ_0'' and

* Received by the PGMTT, August 24, 1961; revised manuscript received, September 20, 1961.

† Thomas J. Watson Research Center, International Business Machines Corp., Yorktown Heights, N. Y.

[1] H. Suhl, "The nonlinear behavior of ferrites at high microwave signal levels," Proc. IRE, vol. 44, pp. 1270–1283; October, 1956. Also, "Theory of ferromagnetic resonance at high signal powers," *J. Phys. Chem. Solids*, vol. 1, pp. 209–227; April, 1957.

[2] R. W. Damon, "Relaxation effects in ferromagnetic resonance," *Rev. Mod. Phys.*, vol. 25, pp. 239–245; January, 1953.

[3] N. Bloembergen and S. Wang, "Relaxation effects in para- and ferromagnetic resonance," *Phys. Rev.*, vol. 93, pp. 72–83; January, 1954.

[4] E. Schlömann, J. J. Green, and U. Milano, "Recent developments in ferromagnetic resonance at high power levels," *J. Appl. Phys.*, vol. 31, suppl., pp. 386S–395S; May, 1960.

[5] P. E. Seiden and H. J. Shaw, "Saturation effects in ferromagnetic resonance," *J. Appl. Phys.*, vol. 31, suppl., pp. 225S–226S; May, 1960.

[6] H. Suhl, "Note on the saturation of the main resonance in ferromagnetics," *J. Appl. Phys.*, vol. 30, pp. 1961–1964; December, 1959.

$1/h$ for each point, plot the data and then extrapolate the $1/h$ behavior back to $\chi''/\chi_0''=1$ to obtain h_c. In this paper we will discuss a new method designed to measure the large signal effect associated with the premature saturation of the normal ferrimagnetic resonance line that will increase the utility of this experiment.

In addition to the saturation of the ferrimagnetic resonance line, there are presently two other types of high-signal experiments which yield information on spinwave linewidths. After describing the new method proposed for the measurements at ferrimagnetic resonance, the advantages of each experiment will be discussed in the light of this method.

II. Experimental Technique

The method we propose records the saturation curve directly in place of point by point measurements so that one can do the extrapolation directly on the recording and obtain h_c from a single calculation. The experiment consists of simply placing the sample in a reflection microwave cavity at a position of RF magnetic field maximum and measuring the power incident on the cavity and power reflected from the cavity by means of crystal detectors. We then use an x-y oscilloscope to display the reflected power[7] as a function of incident power and record the ensuing curve by photographic means. A schematic of the circuitry is shown in Fig. 3. A 2J51 magnetron is used giving peak power outputs of up to 40 kw. This peak power is more than sufficient to obtain microwave magnetic fields of greater than 50 oe. Above this power level the cavity will arc without pressurization or evacuation.

To see how this procedure allows us to determine the critical field for threshold, we will examine the equations describing the cavity absorption and the sample absorption for the conditions at hand. The Q_0 of the cavity is given by

$$Q_0 = \omega_0 \frac{U}{P_D} \qquad (2)$$

where ω_0 is the resonant frequency, U is the energy stored in the cavity, and P_D is the power dissipated in the cavity. The energy stored in the cavity can be written as

$$\omega_0 U = C_1 h^2, \qquad C_1 = \frac{\mu_0\omega_0}{8} \frac{V}{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2} \qquad (3)$$

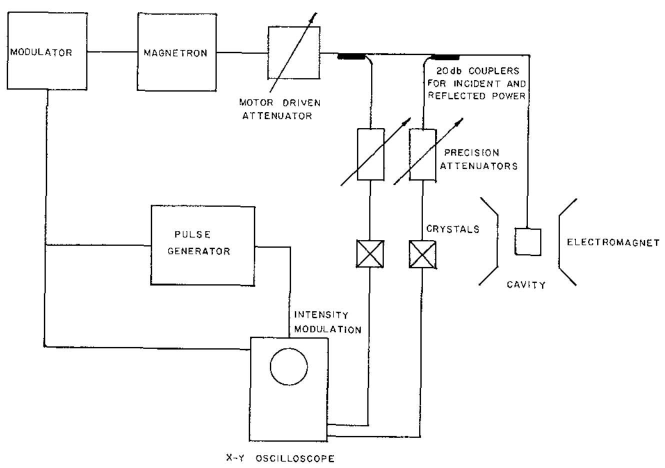

Fig. 3—Circuitry block diagram.

where h is the amplitude of the microwave magnetic field at a position of magnetic field maximum, μ_0 the permeability of free space, V the cavity volume, λ_0 the free space wavelength and λ_c the guide cutoff wavelength. Also, we may write the following relation between the power incident, reflected and dissipated.

$$P_D = P_I - P_R = P_I(1 - |\Gamma|^2)$$

where Γ is the reflection coefficient of the cavity. Substituting for P_D and U in (2), we get

$$h_2 = \frac{Q_0}{C_1} P_I(1 - |\Gamma|^2). \qquad (4)$$

Assuming an undercoupled cavity, we may write

$$Q_0 = \frac{Q_E}{r}$$

where r is the VSWR and is related to Γ by

$$r = \frac{|\Gamma| + 1}{|\Gamma| - 1}.$$

We substitute these relations for Q_0 and Γ in (4) to obtain

$$h^2 = \frac{P_I}{C_1} \frac{Q_E}{(r+1)^2}. \qquad (5)$$

Now we want to consider the effects of the ferrite sample. From perturbation theory for a ferrite sample[8] in a resonant cavity, one finds

$$\chi'' = C_2 \Delta\left(\frac{1}{Q}\right) \qquad C_2 = \frac{1}{4} \frac{V}{v}\left[1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2\right]^{-1},$$

where $\Delta(1/Q)$ is the change in $1/Q$ when one goes from ferrimagnetic resonance to high fields far above reso-

[7] In the experimental arrangement described here, a succession of pulses is used, the amplitude of which is gradually varied by a motor-driven attenuator. Since there is some frequency dispersion on the pulses, spikes will appear on the edges of the reflected pulses due to the fact that these regions contain frequency components outside the bandwidth of the cavity. To eliminate this so that the reflected power at resonance may be used as the ordinate on the oscilloscope presentation, a pulse generator (see Fig. 3) was used to intensity modulate a small part of the pulse containing only the resonant frequency.

[8] J. O. Artman and P. E. Tannenwald, "Measurement of susceptibility tensor in ferrites," *J. Appl. Phys.*, vol. 26, pp. 1124–1132; September, 1955.

nance and v is the volume of the sample. We then have

$$\chi'' = C_2\left(\frac{1}{Q_1} - \frac{1}{Q_2}\right) = C_2\left(\frac{r_1}{Q_E} - \frac{r_2}{Q_E}\right) = C_2\frac{\Delta r}{Q_E}. \quad (6)$$

According to the theory of nonlinear saturation, the susceptibility of the sample will vary as $1/h$ in the nonlinear region. This means that above the threshold we will have

$$\frac{1}{h} = KC_2\frac{\Delta r}{Q_E},$$

where K is a constant. We then substitute for h in (5) to obtain

$$\frac{Q_E^2}{K^2C_2^2(\Delta r)^2} = \frac{P_IQ_E}{C_1(r+1)^2}.$$

If we match the cavity off ferrimagnetic resonance so that it is perfectly coupled, we have

$$\Delta r = r - 1.$$

The additional absorption of the sample on ferrimagnetic resonance serves to undercouple the cavity so that our previous assumption of an undercoupled cavity is not violated, perfect coupling being a limiting case. We may then write

$$\frac{K_2C_2^2P_I}{Q_EC_1} = \left[\frac{r-1}{r+1}\right]^2 = \frac{1}{|\Gamma|^2} = \frac{P_I}{P_R}$$

or

$$P_R = Q_E\frac{C_1}{K^2C_2^2} \quad (7)$$

that is, the reflected power will be a constant as a function of incident power.

At small signals we will have P_R directly proportional to P_I since the susceptibility of the sample is independent of the RF magnetic field. The same will be true of very high signal levels where the sample is saturated since in that region the susceptibility is zero. We should then expect to observe the type of behavior of the reflected power as a function of incident power shown in Fig. 4. If inhomogeneity scattering were not present, the change between the small signal and $1/h$ regions would be abrupt, as illustrated by the dashed lines in Fig. 4. When one does a point-by-point experiment to determine the critical field, as was discussed above, the assumption is made that the linear region observed is that due to the spinwave excitation and that in the absence of inhomogeneity scattering this region would extend back to $\chi''/\chi_0''=1$. The threshold field h_c is then determined by an extrapolation back to $\chi''/\chi_0''=1$ as was shown in Fig. 2. Since the $1/h$ region in Fig. 2 corresponds to the region of $P_R=$const. in Fig. 4, we may make the construction shown by the dashed lines in Fig. 4 to obtain P_c, in analogy with the construction of Fig. 2.

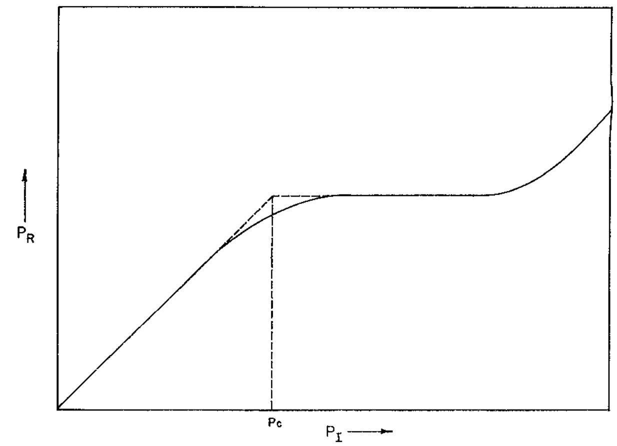

Fig. 4—Saturation curve of reflected power as a function of incident power (schematic).

After one obtains the critical field, ΔH_k is determined from (1), which we rewrite as follows

$$\Delta H_k = 4\pi M_s\frac{h_c^2}{(\Delta H)^2}.$$

Since the nonlinear theory[1] used to obtain (1) implicitly assumes a Lorentzian lineshape, we may write

$$\chi''\Delta H = 4\pi M_s$$

or

$$\Delta H_k = \frac{\chi'' h_c^2}{\Delta H}.$$

Using (5) and (6) to eliminate χ'' and h_c, we have

$$\Delta H_k = \frac{C_2}{C_1}\frac{P_c}{\Delta H}\frac{\Delta r}{(r+1)^2}.$$

The virtue of this equation is that one eliminates the necessity of measuring $4\pi M_s$ and also the cavity, Q. Since χ'' and h^2 depend on Q in an inverse manner, Q drops out of the equation for ΔH_k. One now needs only to measure the linewidth, the VSWR at resonance and the critical power to evaluate ΔH_k.

III. Results

Typical experimentally obtained saturation curves are shown in Fig. 5. The two upper curves show results from two samples whose deviations from the nonlinear theory due to inhomogeneity scattering are greatly different. The almost straight lines at about 55° from the horizontal appearing on the photographs are obtained with a short circuit in place of the cavity. These lines essentially exhibit the small signal behavior and eliminate errors due to different crystal laws. To obtain P_c, one extrapolates the flat regions back to these lines. The upper curves fit the predictions of (7) very well, and it is a simple matter to perform the extrapolations to obtain P_c.

We do not, however, always obtain curves like these,

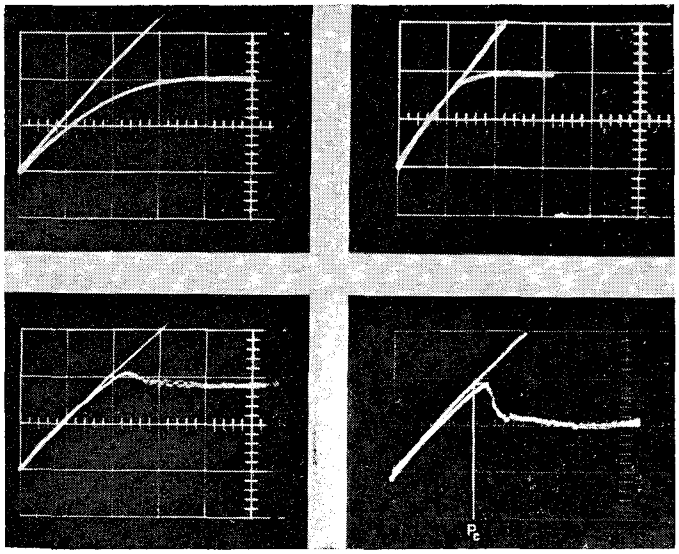

Fig. 5—Typical experimental saturation curves.

and many times a behavior like that shown in the lower curves is observed. This is due to other effects which one encounters at high signal levels.[9] It is observed that under some conditions regions of instability in reflected power occur at suitably high signal levels, accompanied by discontinuous changes in reflected power as a function of incident power. This behavior is what is observed in the lower curves of Fig. 5. However, these additional effects always occur after one has entered the $1/h$ region of the decline of susceptibility; therefore, the small region of peak susceptibility should be usable for the determination of P_c as shown by the construction on one of the lower curves of Fig. 5. This conclusion has been checked experimentally by obtaining h_c both by the new method and by the old method from a point-by-point measurement of the saturation curve,[10] and the results are consistent in every case. The occurrence of these nonideal curves depends on the cavity parameters as well as the sample parameters, indicating that the effect is connected with the coupling between sample and cavity[9] as well as the nonlinear effect itself. Calculations have been carried out which attempted to attribute this behavior simply to the fact that the susceptibility is decreasing linearly with $1/h$, which in turn causes the cavity parameters to change. The calculations do not admit of this type of behavior, thereby indicating that additional nonlinear effects in the sample are important. Since the state of the magnetization above threshold is very complicated, calculations of this effect are extremely difficult. However, regardless of the occurrence of these nonideal saturation curves, it is still possible to determine the critical power and calculate the critical field.

IV. Other Large-Signal Experiments

Of the three types of large-signal experiments presently used, the saturation of the main resonance has been used the least for measurements of ΔH_k. This is for two reasons; first, using the point by point method makes the experiment quite time consuming if one desires to do extensive measurements. Second, the experiment measures only the linewidth of one spinwave k. In the subsidiary absorption experiments[1] where one measures the threshold for the appearance of an absorption at lower dc magnetic fields, and the longitudinal pumping experiment[4,11] where one measures the threshold for nonlinear effects when the dc magnetic field and the RF magnetic field are in the same direction, one may obtain the linewidths of many spinwaves by measuring h_c as a function of dc magnetic field. Also, in these cases, spinwaves are excited where $\omega_k=\omega_0/2$ so that the inhomogeneity scattering into degenerate spinwaves does not affect the nonlinear behavior and the thresholds are sharp.

On the other hand, the experimental technique described here helps alleviate the difficulties caused by the inhomogeneity scattering, but, most important, the range of applicability of measurements made at ferrimagnetic resonance is much greater. For example, the rise time[4] of spinwave effects in longitudinal pumping is quite long, which necessitates the use of pulses in the millisecond range. This means that for samples with large thresholds, heating effects soon become serious and prevent the measurement of the threshold fields. In the case of the main resonance saturation the rise time is quite short and thresholds have been measured as great as 50 oe with no heating effects, since it is possible to use very short pulses. Also, in the case of subsidiary absorption and longitudinal pumping, if the measurement frequency is large compared to $4\pi\gamma M$, the threshold field becomes very high[1,4] and out of the range of measurement.[12] Therefore, working at ferrimagnetic resonance with this new technique allows one to conveniently measure the ΔH_k of the z-directed degenerate spinwave for a large range of materials and frequencies in order to supplement and enlarge the data obtainable with the processes involving $\omega_k=\omega_0/2$ spinwaves.

[9] P. E. Seiden and H. J. Shaw, "High-power effects in ferrimagnetic resonance," *J. Appl. Phys.*, vol. 31, pp. 432–433; February, 1960.

[10] The discontinuities do not appear in the curve of x'' vs $1/h$, but small gaps in the curve do appear, indicating that there exist values of susceptibility which are not stable, and the sample susceptibility jumps across this region as the power is raised.[9]

[11] F. R. Morgenthaler, "Survey of ferrimagnetic resonance in small ferrimagnetic ellipsoids," *J. Appl. Phys.*, vol. 31, suppl., pp. 95S–97S; May, 1960.

[12] If $\omega<\frac{2}{3}\,4\pi\gamma M$, the subsidiary and main resonances are degenerate and since the subsidiary resonance is a lower-order effect, one measures the linewidth of the spinwave associated with this effect. However, one may always carry out threshold measurements at ferrimagnetic resonance, the only difference being that in this case the spinwave of interest satisfies the relation $\omega_k=\omega_0/2$.